STATISTICS

Time: 3 hours   **Marks: 100**

Note: Attempt any five questions, all questions carry equal marks.

Q.1(a) Three married couple have bought six seats in a row for concert. In how many ways can they be seated?

      (i) if all the women sit together   (ii) if all the men refuse to sit together.

(b) A person has purchased 10 out of 1000 tickets sold in a certain raffle. To determine the five prize winners, 5 tickets are to be drawn at random and without replacement. Compute the probability that this person will win no prize.

(c) A class in advance physics is comprised on 10 juniors, 30 seniors and 10 graduate students. The final grades showed that 3 of the juniors 10 of the seniors and 5 of the graduate students received an A for the course. If a student is chosen at random from this class and is found to have earned an A, what is the probability that the student is a junior student?

$$(4+6+10)$$

Q.2(a) Let $f(x_1, x_2) = 6x_2$, $\quad 0 < x_2 < x_1 < 1$, zero elsewhere, be the joint p.d.f. of the random variables X_1 and X_2. Find the conditional mean of X_2 given that X_1.

(b) A manufacturer of small parts sends the parts in boxes of 20 parts to his customers. Assume that each part is either defective or not defective. The probability of an individual part is defective is 0.05. Assume that 10 boxes were produced, what is the probability that there is no defective box in 10 boxes?

$$(12+8)$$

Q.3(a) On an examination the average grade was 74 and the standard deviation was 7. If 12% of the class are given A's and the grades are curved to fallow a normal distribution, what is the highest B?

(b) A production supervisor finds that employees, on the average complete a certain task in 10 minutes. The times required to complete the task approximately normally distributed with the standard deviation of 3 minutes. Find

    i) Percentage of employees requiring more than 5 minutes to complete the task.

    ii) The probability that an employee who has just been assigned the task will complete it within 3 minutes.

(c) Number of persons entering a utility store can be assumed to follow a certain distribution. Over a long period of time it has been established that the number of customers that enter the store in an hour is 60 on the average. Find the probability that in a given two-minute interval no customer will enter the store.

$$(6+8+6)$$

Q.4(a) A taxi company is trying to decide whether the use of radial tires instead of regular belted tires improves fuel economy. Nine cars were equipped with radial tires and driven over a prescribed test course. Without changing drivers, the same cars were then equipped with regular belted tires and driven once again over the test course. The gasoline consumption, in kilometers per liter, was recorded as follows:

Car	Kilometers per Liter	
	Radial Tires	Belted Tires
1	4.2	4.1
2	4.7	4.9
3	6.6	6.2
4	7.0	6.9
5	6.7	6.8
6	4.5	4.4
7	5.7	5.7
8	6.0	5.8
9	7.4	6.9

At the 0.025 level of significance, can we conclude that cars equipped with radial tires give better fuel economy than those equipped with belted tires? Assume the populations to be normally distributed.

(b) The quality control department of a food processing firm claims that the average weight per package of a certain food is at least 25 ounces. Past experience indicates that the standard deviation of weight is 0.3 ounce. A random sample of 144 packages is obtained and the sample mean weight is 24 ounces. Test the claim at $\alpha=0.05$. Also find p-value of the test.

(c) Write down short notes on the following:
    (i) Analysis of Variance   (ii) p-value
    (iii) Systematic Sampling   (6+6+8)

Q.5(a) The following data represent the running times of films produced by two different motion-picture companies:

	Time (minutes)						
Company 1	102	86	98	109	92		
Company 2	81	165	97	134	92	87	114

Test the hypothesis that the average running time of films produced by company 2 exceeds the average running time of films produced by company 1 by 10 minutes. Use a 0.01 level of significance and assume the distributions of times to be approximately normal. Note: first test the equality of variances of time of films.

(b) A geneticist is interested in the proportion of male and females in a population that have a certain minor blood disorder. In a random sample of 100 males, 31 are found to be afflicted, whereas only 24 of 100 females tested appear to have the disorder. Can we conclude at the 0.01 level of significance that the proportion of men in the population afflicted with this blood disorder is significantly greater than the proportion of women afflicted?
                                                             (12+8)

Q.6(a) Explain the terms "Correlation" and describe the properties of the coefficient of correlation. Also define the coefficient of determination.

(b) A study was made by a retail merchant to determine the relation between weekly advertising expenditures (X$) and sales (Y$). The following data were recorded:

        X: 40   20   25   20   30   50   40   20
        Y: 385 400 395 365 475 440 490 420

    i) Plot a scatter diagram.
    ii) Find the coefficient of correlation between X and Y and interpret the result.
    iii) Test that there is no relationship between X and Y.   (6+14)

Q.7(a) Define "Sampling" and discuss the aims of sampling. Also describe the advantages and disadvantages of sampling.

(b) The following table (population) gives the ages (in years) of the five children:

Children:	A	B	C	D	E
Ages (X):	3	5	8	10	11

    (i) Calculate the population mean of ages: μ.
    (ii) Draw all possible samples of size '3' without replacement.
    (iii)Compute the means of the samples (drawn in part-ii) and verify that $\mu = \mu_{\bar{x}}$ where $\mu_{\bar{x}}$ is the mean of the sampling distribution of sample means.
    (iv) Find $\sigma_{\bar{x}}^2$ = variance of the sampling distribution of sample means.

(c) What is Statistical Inference? Explain it with the examples from the field of industry.
                                                                   (5+10+5)